

Derive the Equation $K_a K_b = K_w$

The equation for the ionization of a weak acid HA in solution is: $HA + H_2O \rightleftharpoons H_3O^+ + A^-$

and its K_a expression is: $K_a = \frac{[H_3O^+] [A^-]}{[HA]}$

The equation for the ionization of a weak base A^- in solution is: $A^- + H_2O \rightleftharpoons HA + OH^-$

and its K_b expression is: $K_b = \frac{[HA] [OH^-]}{[A^-]}$

Rearrange the K_a expression above as follows:

$$\frac{[H_3O^+]}{K_a} = \frac{[HA]}{[A^-]}$$

and substitute the left-hand portion into the K_b expression from above to obtain:

$$K_b = \frac{[H_3O^+] [OH^-]}{K_a}$$

Since the equation for the ionization of water is:

$$K_w = [H_3O^+] [OH^-]$$

by substitution and rearrangement, we obtain:

$$K_a K_b = K_w$$

Q.E.D.