The Mathematics of Radioactive Decay

1) Discovery of the radioactive decay law. In 1900, Ernest Rutherford noticed a decrease over time in the radioactive intensity of a sample he was studying, so he began to measure this decrease. He determined that the decrease fit the following formula (using modern symbols):

\[ N = N_0 e^{-kt} \] (1)

where \( N \) = the amount of sample remaining after an amount of time \( t \) and \( N_0 \) = the original starting amount of the sample. \( e \) has its usual meaning and \( k \) is a constant with a unique value for each substance. Rutherford named it the radioactive decay constant.

2) Straight-line form of the decay law. Equation 1 can be modified to the form of a straight-line formula \((y = mx + b)\) as follows:

\[ \frac{N}{N_0} = e^{-kt} \] by rearrangement

\[ \ln \left( \frac{N}{N_0} \right) = -kt \] take natural logarithm of each side (2)

\[ \ln N - \ln N_0 = -kt \] division of exponents is done by subtraction

\[ \ln N = -kt + \ln N_0 \] by rearrangement

This last equation fits the general form of a straight-line equation. The y-axis is the natural log of \( N \) and the x-axis is \( t \) (time). The line will from upper left to lower right (negative slope) with negative \( k \) being the slope.

3) An equation for calculating half-life. The equation (2) can be modified to give an equation where knowing the value for \( k \) leads directly to the length of the half-life.

\[ \frac{-\ln \left( \frac{N}{N_0} \right)}{k} \] by rearrangement

From the definition of half-life, we obtain the value for \( N / N_0 \) as one-half. After one half-life, half the material on-hand at the start will have decayed. If the starting amount equals 2, then the ending amount one half-life later will equal one.

Considering the numerator only: \[-\ln (1/2) = -(\ln 1 - \ln 2) = -(0 - \ln 2) = \ln 2\]

Letting \( T_{1/2} \) = time of one half-life, we obtain:

\[ T_{1/2} = 0.693 / k \] since \( \ln 2 = 0.693 \)

4) Miscellaneous half-life information. The fraction one-half figures prominently in this.

\[ (1/2)^n \] where \( n \) = the number of half-lives yields the decimal portion of substance remaining.

If the decimal fraction of substance remaining is known, setting it equal to \((1/2)^n\) and solving will yield the number of half-lives elapsed.